The Case for Mathematics Coherence

Lessons from Six Districts and the Opportunity for the Field

TNTP

Introduction

Over the past decade, districts across the country have made substantial investments to improve mathematics instruction. States have expanded access to high-quality instructional materials (HQIM), districts have strengthened curriculum implementation, and educators have focused more intentionally on ensuring that all students engage with rigorous, grade-level content. These efforts have created a stronger instructional foundation than existed ten years ago—and they remain essential.

At the same time, the instructional landscape has become more crowded. As systems work to respond to widening performance gaps and unfinished learning, students may encounter a growing array of supplemental programs, intervention resources, assessments, digital platforms, and emerging AI-enabled tools. Many of these products are valuable on their own. But they are often selected at different times, for different purposes, and without built-in connections to one another or to the core curriculum.

That creates a challenge that curriculum adoption alone cannot solve. Students do not experience instructional products in isolation. They move among core instruction, intervention, assessment, tutoring, and technology-based learning opportunities throughout the day. Unless those experiences are intentionally connected, students may encounter a variety of topics and instructional approaches even within the same subject. Districts and educators are then left to the enormous work of translating among products and attempting to build the connections themselves.

TNTP's research has consistently shown why those connections matter. In *The Opportunity Makers*, we found that among nearly 28,000 elementary and middle schools serving students who were, on average, below grade level, only five percent consistently helped students catch up academically over time. One characteristic that distinguished those trajectory-changing schools was a coherent instructional program: supports were organized around a common path toward grade-level learning rather than operating as separate initiatives.

Coherence by Design illustrated what this can make possible. In Tennessee's Knox County Schools, students participating in a coherent instructional model—one that aligned curriculum, diagnosis, grouping, and real-time instructional decisions—averaged 1.3 additional months of learning compared with peers in the district's previous intervention model. The lowest-performing students gained an average of 1.8 additional months in a single school year. *Coherence at Scale* then expanded the conversation beyond districts, showing how state policy, guidance, funding, and accountability can either support or become a barrier to coherent instruction.

Together, this body of work points to the next challenge for the field. Coherence is not only a state and district implementation challenge. It is also a design challenge for the organizations that develop curriculum, assessments, intervention programs, digital tools, and AI-enabled products. Districts have made remarkable progress with what is currently available however educators are faced with the near impossible task of integrating products that were never designed to work together. The next chapter will require products and systems that make the connections among grade-level learning, prerequisite knowledge, assessment evidence, and instructional action easier to see and enact.

The Case for Mathematics Coherence

While coherence matters across subjects, it is especially consequential in mathematics. Mathematics is inherently cumulative: new ideas build on prior understanding, conceptual knowledge develops across years, and representations evolve intentionally as students encounter more sophisticated content. When learning experiences are disconnected, unfinished learning can compound and make future content more difficult to access.

TNTP's *Unlocking Algebra* research illustrates both the challenge and the opportunity. Algebra I is a gateway to advanced mathematics, postsecondary education, and many career pathways. Yet, scores from the 2024 National Assessment of Education Progress (NAEP) show only 28 percent of students nationwide were proficient in eighth-grade mathematics, and TNTP's analysis of more than 2,200 Algebra I students found that 87 percent entered the course without mastering the full set of prerequisite mathematics concepts. Nearly half entered with particularly significant unfinished learning.

Conventional responses often assume that students must revisit every concept they previously missed before they can engage successfully with grade-level mathematics. The *Unlocking Algebra* findings suggest otherwise. Students made the greatest gains when intervention instruction focused on the most important predecessor concepts connected to current grade-level learning. Students who mastered these key predecessors nearly doubled their likelihood of successfully learning new Algebra I content compared with peers who possessed broader but less strategically connected prior knowledge. In simulated instructional models, an individualized approach focused on key predecessors enabled students to acquire substantially more new concepts and skills than either broad remediation or a grade-level-only approach. In other words, what matters is not the amount of prior content students revisit but how intentionally that prior knowledge connects to their current learning.

Accelerate, Don't Remediate reached a similar conclusion in elementary mathematics. Students experiencing a just-in-time acceleration approach were more quickly able to access grade-level content and struggled less than students who began at the same level but received remediation. Across both studies, the implication is clear: effective support does not pull students away from grade-level mathematics. It identifies the specific unfinished learning that matters now and connects that knowledge directly to what students are learning next.

This is also where coherence can reduce cognitive burden. Mathematics is challenging enough without requiring students to reconcile conflicting representations, routines, and expectations across settings. Educators likewise should not have to spend scarce planning time translating among disconnected products and manually reconstructing a learning trajectory. When curriculum, intervention, assessment, and instructional supports align around shared goals, students can focus on making mathematical connections and educators can focus more fully on teaching.

Unlocking Algebra therefore does more than make the case for acceleration. It reveals a central design need for mathematics. Educators need timely, precise information about which prerequisite knowledge is most important, how it connects to current grade-level content, and what instructional response is most likely to move learning forward. They also need the resources to provide those responses. Most existing products do not yet provide that full chain of instructional actionability. That gap became a central question in TNTP's six-district study.

If coherence matters so much in mathematics, what do students experience as they move among core instruction, intervention, assessment, and technology-based learning opportunities? To what extent do instructional materials, supplemental supports, assessments, professional learning, and other system components reinforce a common mathematical storyline, shared expectations, and grade-level learning goals? Where do the connections begin to break down, and what can the field learn from the ways districts are already trying to build them?

Learning from Six Districts

To explore these questions, TNTP partnered with six districts that had already made significant investments in mathematics instruction including core curriculum, organizational structures, and professional learning. Each district identified a small group of elementary and/or middle schools where leaders believed promising practices were emerging. The goal was not to rate districts or identify a single model. It was to understand how instructional products, educator practices, organizational systems, and district structures interacted, and to identify recurring patterns that could inform the field.

Study Design

The study combined district document and product review, classroom observation, professional learning community observations, focus groups and interviews, and limited quantitative analysis. Researchers reviewed core curricula, intervention programs, assessment systems, MTSS and RTI guidance, mathematics scope and sequence documents, professional learning resources, technology-based tools, and district mathematics vision documents.

Researchers paid particular attention to how core instructional materials connected to intervention resources, assessment systems, and digital tools. In many districts, these components had been selected or implemented at different times and for different purposes. The study examined whether educators could use them as part of a connected mathematics pathway or whether they functioned as separate instructional experiences requiring educator capacity and expertise to bring together

Site visits provided a complementary view. Researchers observed Tier I, Tier II, and Tier III mathematics instruction; attended collaborative planning and PLC meetings; and spoke with teachers, interventionists, instructional coaches, school leaders, and central-office staff. District documents revealed the formal systems intended to support instruction. Classroom observations and educator conversations revealed how those systems operated in practice, where connections were visible, and where educators had to bridge gaps themselves.

Looking for Patterns, Not Models

The study was not designed to prove that any participating district had achieved coherence or to establish causal links between particular practices and student outcomes. Instead, researchers looked across multiple evidence sources for recurring ways districts were working toward more connected mathematics experiences. Two patterns became especially visible. They are presented here as archetypes, not prescriptions or rankings, but as a way to understand the progress districts have made, the limits they encountered, and the product design opportunities those limits reveal.

Assumptions That Guided Our Research

- **High-quality instructional materials are the foundation, but they are not enough**.
  Without a strong core, there is no common mathematical pathway to connect; even with one, the full instructional stack (curriculum, assessments, intervention, professional learning, and AI-enabled tools) must still align around it.
- **Students do not need to relearn everything they have missed before they can succeed in grade-level mathematics.**
  They need targeted support with the key predecessor knowledge that enables access to current grade-level learning.
- **Students experience coherence, but systems create it.**
  Materials, schedules, professional learning, collaboration structures, assessments, and instructional decisions determine whether learning experiences reinforce or compete with one another.
- **More resources do not automatically create better outcomes**.
  Adding programs without integrating or simplifying the existing system can increase complexity and shift the burden of coherence to educators.

These assumptions shaped our research from the outset. Rather than focusing on individual programs or instructional components, we examined how districts connected core instruction, supplemental instruction, assessment, professional learning, technology-based resources, and the organizational structures that support them. We also explored how educators coordinated these elements across instructional settings to create coherent learning experiences that supported access to grade-level mathematics.

Archetype 1: Building from a Strong Instructional Foundation

The first pattern reflects districts that invested deeply in the instructional program itself, particularly through the use of high-quality curriculum. Their theory of action was straightforward and powerful: when educators share a common vision for mathematics, supported by strong materials, common learning progressions, shared language, and sustained professional learning, students are more likely to experience mathematics as a connected discipline.

Defining Characteristics

In these districts, curriculum implementation served as the anchor for many other instructional decisions. Materials adoption was not treated as a one-time event. Leaders continued to invest in professional learning that deepened educators' understanding of the mathematics embedded in the materials and the instructional practices needed to bring it to life. Importantly, some of this learning extended across roles and settings. In some cases, interventionists as well as classroom teachers developed familiarity with core routines, representations, and progressions.

This shared foundation produced greater consistency across classrooms and, in the strongest examples, across core and supplemental instruction. Educators had a common reference point for what students were learning, how ideas developed over time, and what high-quality grade-level instruction should look like. For these districts, HQIM implementation was the foundational first step, the bedrock from which broader coherence could grow.

What This Looked Like in Practice

Students regularly encountered grade-level mathematics grounded in strong instructional materials. Across classrooms, educators used common mathematical language, familiar representations, and predictable instructional routines. These consistencies provided the opportunity for students to connect new learning to their growing understanding of mathematics.

The influence of the core sometimes extended into intervention. Where classroom teachers, interventionists, and coaches shared professional learning and planning routines, supplemental instruction was more likely to preview upcoming concepts, revisit specific prerequisite ideas, or use the same models and language students encountered in Tier I. Strong implementation of the core created a gravitational pull toward greater coherence across settings.

But that connection was not guaranteed. Intervention programs, assessments, and supplemental tools still varied, and the work of aligning them often fell to individual educators. Some teams successfully translated assessment evidence into targeted support connected to current learning. Others relied on broader indicators of performance that made it difficult to identify the specific prerequisite concepts students needed. In those settings, intervention became an additional experience rather than an extension of the grade-level pathway.

Key Strengths

Districts in this pattern demonstrated several notable strengths that created a strong instructional foundation for mathematics:

- Sustained investment in strong core materials and curriculum implementation, including professional learning focused on both mathematical content and pedagogy.

- A shared instructional language, set of representations, and predictable routines that reduced unnecessary variation for students and was anchored in HQIM.

- A classroom-level commitment to mathematics as a coherent progression, supported by materials that helped educators connect new ideas to prior concepts and maintain grade-level expectations.

Common Challenges

Much of the coherence researchers observed still depended on the expertise, collaboration, and initiative of individual educators. There was variability in how intervention connected to core instruction, how assessments supported day-to-day decisions, and how schools identified the unfinished learning most relevant to current content. The instructional foundation was strong, but the mechanisms connecting curriculum, intervention, assessment, and instructional action were less consistently designed.

Perhaps the best metaphor for this archetype is that of a partially completed puzzle. The central image and many of the strongest pieces are in place: a common vision for mathematics, a strong core curriculum, and shared educator learning. But assessments and supplemental resources do not always fit neatly around that foundation, and the final instructional and systemic connections still require skilled, time-consuming local work.

These districts do not primarily need more programs. Additional curriculum, intervention products, or assessments alone are unlikely to solve the challenge. The opportunity lies both in helping existing components work together more intentionally and in designing product features that make the connections easier to enact. Curriculum, intervention, assessment, and technology tools could support coherence more directly by making links to grade-level learning explicit, identifying the most consequential unfinished learning, and clarifying how instructional decisions across settings reinforce a common mathematics trajectory.

Archetype 2: Educators as Skilled Integrators

In the second pattern we identified, districts worked deliberately to create coherent mathematics experiences by connecting instructional resources, educator practices, and organizational systems around a shared vision for student learning. Because the curriculum, assessments, intervention resources, and technology available to schools did not always connect seamlessly, district and school teams invested significant time, expertise, and organizational capacity to bridge these gaps.

Defining Characteristics

Leaders in these districts often found that the curriculum, assessments, intervention resources, and technology tools provided important pieces of the instructional system but were not designed to work together in ways that fully supported their vision for coherent mathematics learning. In response, districts built a local integration layer around those resources. They created detailed guidance, cross-role planning structures, coaching routines, data discussions, intervention frameworks, and leadership processes designed to connect instructional decisions across settings and roles.

Cross-functional collaboration was central to this approach. The focus was on classroom teachers, interventionists, coaches, school leaders, and district staff coordinating their work rather than operating as separate service providers. District-developed documents and routines attempted to connect curriculum, assessment, intervention, and instructional decision-making in ways the resources themselves did not yet consistently support.

What This Looked Like in Practice

Researchers observed professional learning communities that moved beyond reviewing scores to discussing instructional responses. Interventionists participated in planning with classroom teachers. Coaches helped teams trace mathematical ideas across grade levels and consider how supplemental instruction could prepare students for current or upcoming content. Leadership meetings and coaching cycles focused on coordinating decisions across roles and settings.

Assessment evidence sometimes became a shared input into these conversations. Teachers, coaches, and leaders examined student work and multiple data sources to identify unfinished learning and decide how to respond. Yet even highly collaborative teams often lacked a single tool that moved seamlessly from assessment, to diagnosis, to identification of key prerequisites, to a precise instructional response. Educators pieced together evidence from multiple systems and used professional judgment to create the missing connections.

This work reflected sophistication, not failure. Districts understood the coherence challenge and developed extensive local guidance to compensate for it. Their strongest structures made instruction more responsive and helped adults share knowledge of student progress. But the time and expertise required were substantial, and the quality of the resulting connections still depended on local mathematics knowledge and leadership capacity.

Key Strengths

- Collaboration was intentional rather than incidental, with protected structures for educators across roles to examine mathematics, student learning, and instructional responses together.
- District guidance and leadership routines helped educators navigate complexity and connect resources that otherwise operated independently.
- Responsibility for student learning was shared across classroom teachers, interventionists, coaches, school leaders, and central-office teams.

Common Challenges

Strong organizational systems did not automatically produce mathematical coherence. Teams could meet regularly and communicate effectively, and yet students still encountered different language, representations, learning progressions, or expectations across classrooms and intervention settings. Collaboration helped compensate for product fragmentation, but it could not eliminate it.

The puzzle metaphor is different here. Imagine opening six puzzle boxes, each with a different picture on the cover, then being asked to assemble one complete image from all of the pieces. That was the challenge these districts faced. They trimmed one piece, translated another, and created connectors that did not come in the box. Curriculum, assessments, benchmarks, intervention materials, state guidance, supplemental tools, and digital products each offered useful pieces, but none had been designed as part of the same puzzle.

Even the most skilled integrators faced limits. Many districts lacked tools that could consistently identify which students needed additional support, pinpoint the unfinished learning most consequential for grade-level access, monitor whether intervention was accelerating progress, and recommend the next instructional adjustment. This is the challenge Unlocking Algebra makes visible: mathematics educators need actionable knowledge of learning progressions and key predecessors, not simply more data.

This pattern suggests that coherence remains as much a design challenge for the field as an implementation challenge for districts. Human relationships, expertise, and collaboration will always matter. But they should not have to carry the full burden of making independently designed instructional products function as a coherent mathematics system. There is only so much educators can do when the pieces were not designed to work together.

From Research to the Mathematics Coherence Framework

Across the six districts, researchers observed two meaningful starting points. Some systems built outward from a strong instructional foundation grounded in HQIM, common progressions, and sustained educator learning. Others built an integration layer of guidance, relationships, and organizational routines around a more diffuse product landscape. Neither pattern should be understood as a finished model, and neither alone addressed every dimension of mathematics coherence.

Districts with strong instructional foundations still struggled to connect intervention, assessment, and instructional decision-making with enough precision. Districts with strong coordination structures still had to reconcile materials and tools that used different mathematical language, progressions, and expectations. Across both patterns, the most difficult frontier was instructional actionability: moving from evidence of unfinished learning to a precise understanding of what matters most and then to a coherent instructional response connected to grade-level content.

These findings shaped the TNTP Mathematics Coherence Framework. The framework is not intended to prescribe one program or organizational model. It helps cross-functional teams examine whether the full instructional system supports a connected student experience including whether progressions align, whether language and representations remain consistent, whether assessment information leads to actionable decisions, and whether organizational structures make those connections durable rather than dependent on individual effort.

The framework also points to a broader call to action. Districts have done remarkable work implementing high-quality materials and creating local coherence with the products available today. They should continue strengthening those systems. But the next opportunity does not belong to districts alone. Curriculum developers, assessment providers, intervention partners, dashboard designers, and technology and AI developers can make coherence more possible by designing products around a shared mathematical storyline and a more complete chain of instructional decision-making.

The field is beginning to move in this direction. Emerging products are exploring tighter connections among curriculum, assessment evidence, prerequisite knowledge, and instructional recommendations. That progress is encouraging, but it is still early. The next generation of mathematics tools should help educators move more seamlessly from assessment to diagnosis, from diagnosis to the key predecessor knowledge that matters, and from that knowledge to a grade-level instructional response—without asking every district to build the connective tissue from scratch.

Coherence is experienced by students, created by systems, and increasingly enabled, or constrained, by product design. High-quality instructional materials established the foundation. The next chapter is to ensure that every component surrounding those materials helps students experience mathematics as one connected learning journey toward grade-level success.

About TNTP

TNTP is the leading research, policy, and practice organization dedicated to transforming America's public education system to meet the needs and expectations of tomorrow—for students, families, communities, and the nation. We are educators, researchers, innovators, and futurists committed to the belief that every student deserves access to a quality education and formative experiences that lead to a lifetime of economic and social mobility, regardless of zip code, race, class, primary language, or learning style.

References

2022 National Assessment of Educational Progress Long-Term Trend Assessment Report: https://www.nationsreportcard.gov/highlights/ltt/2022/

Accelerate Don't Remediate: New Evidence from Elementary Math Classrooms. TNTP. (2021). https://tntp.org/publication/accelerate-dont-remediate/

Coherence by Design: Building Instructional Systems that Help Students Catch Up. TNTP. (2026). https://tntp.org/publication/coherence-by-design/

Coherence at Scale: A State Guide to Supporting Instructional Systems. TNTP. (2026). https://tntp.org/publication/coherence-at-scale/

The Opportunity Makers: How a Diverse Group of Public Schools Helps Students Catch Up -- and How Far More Can. TNTP. (2024). https://tntp.org/publication/the-opportunity-makers/

Unlocking Algebra" What the Data Tells us About Helping Students Catch Up. TNTP. (2025). https://tntp.org/publication/unlocking-algebra/